

Shot noise:

What is the electronic current shot noise?

Because of the discrete charge nature of the electron, the flow of current at any given point in time will actually be grainy and fluctuates from time to time. This fluctuation of the current around the mean is the shot noise and it is intrinsically related to the electron's charge. The flow of current is a Poissonian process, since the passage of electrons at a given point in the circuit is random in the same manner as rain drops falling into a bucket

An intuitive way to understand the relationship between the electron's charge and shot noise is to consider the case of an imaginary rain. Suppose the average rate of water falling from the sky into a bucket is constant, say 1 liter per minute. And let's further assume that the raindrop size is uniform. In the case where each raindrop's volume is a liter, we would expect an average of 1 raindrop per minute. But, the raindrop distribution is Poissonian. So we can expect to see 0 raindrop in a minute sometimes, 1 occasionally, and 2, 3, 4.... etc. From our understanding of Poissonian statistics, we know that the stdev of raindrop per minute is 1 and therefore the stdev of rainfall rate to be 1 liter per minute. So, we expect the rainfall rate to appear to vary significantly from time to time; the large size of each raindrop makes the rainfall appear 'grainy'. On the other hand, if the raindrop's volume is a c.c., we can expect an average of 1000 raindrop per minute. The stdev of raindrop per minute is then 33 and correspondingly the stdev of water falling rate is .033 liter per minute. The rainfall rate, in this situation, is therefore more uniform. In the limit where the raindrop is infinitesimally small, the rainfall rate will be perfectly uniform and smooth in time. For the case of current flowing in a circuit, the charge of the electron is similar in interpretation to the raindrop's volume discussed in this example.

Let's do a little math to flesh this out. The way to estimate the current flow (which have a mean of I) will be to measure the amount of charge Q that crosses a given point in the circuit in a given amount of time t . An estimate for the current flow is given by Q/t . Q will be equal to the number of electrons detected multiplied by the electron's charge, e . On average, we will expect this number to be equal to It/e . But, of course, this number is going to vary from one measurement to another. In fact, the stdev is $\sqrt{It/e}$ (Poissonian). So, we get

$$I_{estimate} = Q/t = I \pm \sqrt{Ie/t} . \quad (1)$$

The larger charge e is, the greater the fluctuation. By measuring the current over a longer time t , we can suppress the fluctuation to some extent. This fluctuation is the shot noise.

There is another way to express shot noise. In the frequency domain, the shot noise is given by:

$$\sqrt{Ie(NEB)} , \quad (2)$$

where NEB is the acceptance frequency bandwidth.

We can see a connection between the two formulas. The frequency transform of a time window of width t is a sinc function. The nominal frequency bandwidth of the sinc function is $1/t$.

The shot noise described in Eq. (2) is the shot noise around zero frequency (we are looking for an estimate of the dc component of I) with an acceptance frequency bandwidth that is characteristic of the time window duration.

But, Eq. (2) is more generally applicable. The shot noise around any frequency f is given by the same result. We can see this in the following manner. We have established that the electronic current follows a Poissonian statistics distribution. And we have seen from our discussion on the Poissonian statistics that its noise spectrum is white (spectrally flat). Therefore, the calculation for the noise component at a frequency window at any arbitrary center frequency will be identical to the calculation at zero center frequency.

An excellent reference source for a more formal look at shot noise can be found in:
<http://ocw.mit.edu/NR/rdonlyres/Physics/8-13Experimental-Physics-I---II--Junior-Lab-Fall2002/5E5023FB-36FF-4B59-A2B8-CD8DD88456F1/0/jlexp43.pdf>